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# Symmetry Relations for Deep Inelastic Processes*

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Relations between structure functions conventionally obtained from quark-parton or light-cone models are shown to follow from model-independent symmetry assumptions common in hadron scattering. Exotic t-channel exchanges are forbidden. No strong-interaction symmetry beyond isospin is assumed and SU(3) is used only for vertices involving currents. Relations originally derived for nucleon targets hold for any isospin mirror pair and apply to complex targets appearing in the Mueller formalism for inclusive processes in the target fragmentation region. New relations are derived.

Numerous relations between the electromagnetic and weak structure functions have been obtained[1] by use of either the quark-parton model or the light-cone algebra. We wish to point out that (1) many of the relations can be obtained from general symmetry conditions and are present in a wide class of models,[2] and (2) the same general conditions can be applied to inclusive reactions and lead to new sets of relations. Consider, for instance, the relations[1]

$$4 \geq \frac{F_1^{\gamma H}}{F_1^{\gamma \widetilde{H}}} \geq \tfrac{1}{4}, \tag{1a}$$

$$F_1^{\gamma H} + F_1^{\gamma \widetilde{H}} \geq \tfrac{5}{18} \left[ F_1^{\nu H} + F_1^{\nu \widetilde{H}} \right], \tag{1b}$$

where H and $\widetilde{H}$ are isospin mirror states. These relations deal with the dependence of the scaling function $F_1(\omega = 2M\nu/Q^2)$ on internal symmetry variables alone at fixed values of the energy-momentum variables. They were originally derived for proton and neutron targets, but can be generalized to any pair of isospin mirror targets including complexes of particles. Furthermore, they can be derived under assumptions automatically included in conventional descriptions of the scattering of any SU(3) octet of bosons on any hadron target. All that is required is (1) the absence of exotic t-channel exchanges, (2) good SU(3) properties for the vertex couplings of the bosons to the t-channel exchanges, but no symmetry assumptions regarding the hadron target, except for charge symmetry or isospin, and (3) equality of the vector-vector and axial-axial contributions to the structure function $F_1(\omega)$.

Pallua and Renner[3] have derived Eq. (1a) by use of conventional strong-interaction assumptions and have also investigated consequences of duality.[4] However, our assumption (2) is significantly weaker than theirs and our results are therefore unaffected by SU(3) symmetry breaking in strong interactions and can easily be extended to arbitrary hadron targets.

This discussion leads immediately to the following observations. (1) Any disagreement between such relations and experiment will do much more than throw out the naive quark-parton or light-cone models. It will automatically disprove any model that satisfies the underlying symmetry conditions that are sufficient to obtain the relations. (2) Any relation derived for the nucleon form factors holds also for the corresponding form factors for <u>any pair of hadrons</u> that satisfies the symmetry requirements used to obtain the relations. For example, relations (1) hold for <u>any isospin mirror pair</u>, e.g., $\pi^+$ and $\pi^-$. They therefore do not probe any details of hadron structure beyond charge symmetry. The assumptions that are sufficient to give the relations (1) are precisely the following.

1. The conventional SU(3) transformation properties of the electromagnetic and weak currents. The electromagnetic current is the U-spin scalar component of an octet; the weak vector currents are members of the <u>same</u> octet.

2. The conventional normalization of weak vector relative to electromagnetic vector contributions and of axial vector relative to vector contributions. This is needed for relations (1b) but not for (1a).

3. No exotic t-channel exchanges. In a current-commutator description, the relevant piece of the commutator of the octet currents at two different space points (e.g., the piece near the light cone or the piece that contributes to parton matrix elements) transforms under SU(3) like some combination of octet and singlet. In a Regge description, there are no exotic Regge exchanges.

4. SU(3) relations for the couplings between currents and any exchange with definite t-channel quantum numbers. No strong-interaction symmetry higher than isospin is assumed for the hadron vertex.

That these assumptions are indeed sufficient is shown for some specific cases. A more complete treatment will be presented elsewhere. Consider the structure function $F_1(\omega)$ for the forward Compton amplitude of currents on hadrons, i.e., for reactions of the form

$$b + \beta \rightarrow a + \alpha, \tag{2}$$

where the Greek letters specify hadrons and Latin letters specify currents. The general SU(3) structure of the amplitude, subject to assumptions (1), (2), and (3), is given by

$$(F_1)^{ab}_{\alpha\beta} = i\, f^{abc} F^c_{\alpha\beta} + d^{abc}\, D^c_{\alpha\beta} \ , \tag{3}$$

where $f^{abc}$ and $d^{abc}$ are the usual SU(3) structure constants. The parameters $F^c_{\alpha\beta}$ and $D^c_{\alpha\beta}$ are not specified by the theory and describe the dependence of the amplitude on the t-channel quantum numbers and on the hadron states $\alpha$ and $\beta$. The structure functions for incident photons and neutrinos are

$$\left[F_1^{(\gamma)}\right]_{\alpha\beta} = \tfrac{1}{6}\, D^3_{\alpha\beta} + (1/6\sqrt{3})\, D^8_{\alpha\beta} + \tfrac{1}{3}(\sqrt{2}/\sqrt{3}) D^0_{\alpha\beta}, \tag{4}$$

$$\left[F_1^{(\nu)}\right]_{\alpha\beta} = -F^3_{\alpha\beta} + (1/\sqrt{3})\, D^8_{\alpha\beta} + (\sqrt{2}/\sqrt{3})\, D^0_{\alpha\beta}. \tag{5}$$

Equations (1) follow directly from (4) and (5) on setting $\alpha = \beta = H$ or $\widetilde{H}$. In this derivation, only the positivity of $D^c_{\alpha\beta}$ is used—not the full content of the positivity of the matrix $(F_1)^{ab}_{\alpha\beta}$. Furthermore, only charge symmetry is required—not the full SU(2).

A convenient mathematical trick which gives results exactly equivalent to Eqs. (3—5) is to write the scattering amplitude for the process (2) in terms of the Levin-Frankfurt additive quark model[5] for high-energy scattering. The current is written as the particular linear combination of quark-antiquark pairs that carries the SU(3) quantum numbers of the current. The scattering of the current on a hadron H is written as the sum of all possible terms in which only a single quark or antiquark from the current is scattered on the hadron and the other antiquark or quark remains a spectator. This well-defined mathematical procedure does not assume anything about the existence of physical quarks, but expresses the observed amplitude in a convenient form which is also the most general form with SU(3) couplings at the boson vertex and no exotic exchanges.

In order to generalize the previous results to inclusive reactions,[6] we consider the process

$$J_\mu + B \to C + X, \tag{6}$$

where $J_\mu$ is the incident current, B is the target, and C is the detected particle. The cross section for such a process is given by[7]

$$\frac{d\bar{\sigma}}{dQ^2 d\nu d\Gamma} = \frac{4\alpha^2}{Q^4} \frac{E'}{E} \frac{Q^2}{\nu} \left(1 - \frac{Q^2}{2M\nu}\right)\left[\frac{d\sigma_S}{d\Gamma} + \frac{E'}{2E} \frac{d\sigma_R}{d\Gamma} + \frac{E}{2E'} \frac{d\sigma_L}{d\Gamma}\right] \quad (7)$$

where S, R, and L denote the scalar, right-handed, and left-handed polarization of the current, respectively, $d\Gamma$ is a phase-space factor for the detected particles, and $\bar{\sigma}$ denotes the cross section averaged over the azimuthal angle $\phi$ between the hadronic system and the lepton plane. When the detected particle has spin zero, then

$$d\sigma_R/d\Gamma = d\sigma_L/d\Gamma\,. \quad (8)$$

Furthermore, the relation

$$\int (d\sigma_S/d\Gamma)d\Gamma = \langle n\rangle\, \sigma_S \quad (9)$$

indicates that the vanishing of $\sigma_S$ implies vanishing of $d\sigma_S/d\Gamma$. In the target fragmentation region, the above reaction can be presented in the Mueller formalism[7] as the forward scattering of a current on a complex $B\bar{C}$. The previous assumptions concerning the vertices can be applied again to give the following results.

1. The absence of t-channel exotics, in particular the absence of I = 2 exchanges, leads to

$$2\sigma_i(J + d \to \pi^0 + x) = \sigma_i(J + d \to \pi^+ + x) + \sigma_i(J + d \to \pi^- + x), \quad (10)$$

where J is any weak or electromagnetic current. This result holds also when the pion is replaced by any isovector hadron, e.g., by a $\Sigma$ or $\rho$.

2. Relations (1) are immediately valid for any model that makes the four assumptions listed above. Thus we obtain

$$4\sigma_T(\gamma B \to CX) \geq \sigma_T(\gamma\widetilde{B} \to \widetilde{C}X) \geq \tfrac{1}{4}\,\sigma_T(\gamma B \to CX), \tag{11a}$$

$$\sigma_T(\gamma B \to CX) + \sigma_T(\gamma\widetilde{B} \to \widetilde{C}X) \geq \tfrac{5}{18}\left[\sigma_T(\nu B \to CX) + \sigma_T(\nu\widetilde{B} \to \widetilde{C}X)\right], \tag{11b}$$

where $\sigma_T$ denotes the transverse cross section and B and $\widetilde{B}$ and C and $\widetilde{C}$ are any isospin mirror pairs. For example, the targets B and $\widetilde{B}$ can be proton and neutron or can both be deuterons. The produced particles C and $\widetilde{C}$ can be p and n, n and p, $\pi^+$ and $\pi^-$, $\pi^-$ and $\pi^+$, $K^+$ and $K^0$, or $K^0$ and $K^+$. Note that a new relation results from interchanging C and $\widetilde{C}$ while B and $\widetilde{B}$ are kept unchanged and different; e.g., $\pi^+$ production off protons is related to $\pi^-$ production off neutrons, and $\pi^-$ production off protons is related to $\pi^+$ production off neutrons. These relations illustrate the power of our general symmetry approach. Note that in the case in which B and C are a nucleon and pion, the $B\overline{C}$ system is a mixture of isospins $\frac{1}{2}$ and $\frac{3}{2}$ and a mixture of SU(3) representations 8, 10, $\overline{10}$, and 27 and is not easily treated by the methods of Refs. 3 and 4, which assume an octet target.

3. The results (10) and (11) can be combined to give the inequalities

$$4 \geq \frac{\sigma_i(\gamma p \to \pi^+ X) + \sigma_i(\gamma p \to \pi^- X)}{2\sigma_i(\gamma p \to \pi^0 X)} \geq \tfrac{1}{4}\,, \tag{12a}$$

$$\tfrac{5}{2} \geq \frac{\sigma_i(\gamma d \to \pi^0 X)}{\sigma_i(\gamma d \to \pi^\pm X)} \geq \tfrac{5}{8}\,. \tag{12b}$$

4. The consequences of the absence of I = 2 exchanges for targets and particles with arbitrary isospin are easily seen by applying the quark trick described above to inclusive reactions. The

inclusive cross section for the reactions (2) can be expressed in the fragmentation region of the target as the sum of contributions having the form of cross sections for the reactions

$$q + B \to C_M + X, \tag{13}$$

where q is either a quark or an antiquark, and the subscript M labels the eigenvalue of $I_z$. The dependence of the inclusive cross section $\sigma_M$ for the process (13) on the charge of the state $C_M$ is a given isospin multiplet is shown by Peshkin's theorem[8] to have the form

$$\sigma_M = \sum_{M=0}^{2I_B+1} a_n M^n. \tag{14}$$

That is, it is a polynomial in M and is of degree $2I_B + 1$, where $I_B$ is the isospin of the target B. For a deuteron target, $I_B = 0$ and $\sigma_M$ is a linear function of M, as in Eq. (10). For a nucleon target, $I_B = \frac{1}{2}$ and $\sigma_M$ is a quadratic function of M. The upper limit is $2I_B + 1$, rather than $2I_B + 2$ because of the quark trick. The difference results from adding the constraint of forbidding I = 2 exchange to the constraints following from isospin invariance alone.[9] Straightforward application of Eq. (14) to specific cases gives many relations, such as Eq. (10) and the relations

$$\sigma(Jd \to \Delta_M X) = a + bM, \tag{15a}$$

$$\sigma(JN \to \Delta^{++}X) - \sigma(JN \to \Delta^{-}X) = 3[\sigma(JN \to \Delta^{+}X) - \sigma(JN \to \Delta^{0}X)]. \tag{15b}$$

The inequalities that can be obtained for inclusive pion production by neutrinos are somewhat stronger than those given by Llewellyn-Smith and Pais.[9] Adding to their analysis the additional constraint of the quark trick or the equivalent condition of no I = 2 exchange gives

$$6\sigma(\nu p \to \pi^+ X) + 2\sigma(\nu p \to \pi^- X) \geq 6\sigma(\nu p \to \pi^0 X) \geq 3\sigma(\nu p \to \pi^- X). \qquad (15c)$$

The derivations of all the consequences so far have utilized the positivity of $F_1$ only. The stronger positivity relations $F_\pm = F_1 \pm \frac{1}{2} F_3$ have not been used at all, though they are satisfied in the quark-parton and in the light-cone model. Combining them with the most general positivity conditions for inclusive processes[10] gives

$$\frac{F_1(\gamma p \to \pi^0 X)}{F_1(\gamma p \to \pi^- X)} \geq \frac{1}{3}, \quad \frac{F_1(\gamma p \to \pi^+ X)}{F_1(\gamma p \to \pi^- X)} \geq \frac{1}{8},$$

$$2 + \sqrt{3} \geq \frac{F_1(\gamma p \to \pi^0 X)}{F_1(\gamma p \to \pi^+ X)} \geq \frac{1}{8}. \qquad (16)$$

The internal symmetry relations for deep inelastic scattering thus follow from a set of assumptions that are conventional in strong interactions and that seem to be in good agreement with experiment in purely hadronic processes and in electromagnetic processes that test the SU(3) classification of the photon. It therefore seems reasonable to search for additional tests in the deep inelastic region where they are expected to hold for all values of $\omega$, and to look for other tests to distinguish between the different models having the same underlying symmetry structure.

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